Positron annihilation in flight

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In this resource article, an exceptional bubble chamber picture—showing the annihilation of a positron (antielectron e^+) in flight—is discussed in detail. Several other esoteric phenomena (some not easy to show on their own!) also manifest themselves in this picture—*pair creation* or the *materialization* of a high energy photon into an electron–positron pair; the 'head-on' collision of a positron with an electron, from which the mass of the positron can be estimated; the *Compton Effect*; an example of the emission of electromagnetic radiation (photons) by accelerating charges (*bremsstrahlung*).

It is hoped that this article can be useful on several levels.

• Bubble chamber pictures demonstrate in a believable way the 'reality' of esoteric phenomena taking place in a few billionths of a second. The pictures also have a mysterious beauty.

• Elementary physics concepts—such as Newton's Second Law, momentum and energy conservation, the forces exerted by magnetic fields on electric currents (moving charges), the fact that the origin of electromagnetic radiation is accelerating charges—are seen to be relevant in the exotic interactions created in particle physics.

These same interactions played a crucial part in the early universe. If particles did not have the properties they do, atoms as we know them would not exist. Neither would we!

Such considerations show how important an appreciation of basic science is to our culture, and should be included in our science education. It is also important to the self-esteem of young people to know that what they are studying has a deep connection with the meaning of human life.

• Many teachers and bright students visiting the School of Physics and Astronomy at the University of Birmingham have commented that they would like detailed discussions of particle interactions, to consolidate their reading of popular books and articles. Some, sadly, even feel somewhat alienated by popularizations.

There is something very satisfying about seeing how straightforward relativistic kinematics describes particle phenomena. However, the algebra, although made up of simple steps, is sometimes rather longer than is usual at A-level. Here, every step is explained.

• The material presented here could be used in conjunction with various first-year university courses.

• Several problems are set and solutions provided (or referenced).

The article is planned as follows:

1. What is particle physics?
2. The bubble chamber.
3. A detailed qualitative description of a variety of features of one exceptional bubble chamber picture.
4. Summary of relevant relativistic kinematics.
5. A detailed discussion of positron annihilation in flight.

What is particle physics?

The aim of particle physics is to study the fundamental building blocks of nature and the forces they exert on each other. The experimental side of this subject consists of examining what happens when particles are made to collide at high energies at *accelerator* centres such as CERN (the European Laboratory for Particle Physics) in Geneva and Fermilab near Chicago. The particles emerging from such collisions are recorded in instruments called *detectors.*

In this article, we consider a side-show, and discuss in isolation a small part of the final state of a high energy interaction, in a bubble chamber, between a neutrino and a neutron or proton inside a neon nucleus.

The bubble chamber (as an example of a detector)

If two aeroplanes with vapour trails behind them were to approach each other, circle around, and then go their separate ways, the fact that they had done so would be apparent for quite a while. A permanent record of the encounter could be obtained by taking a photograph of the vapour trails.

A particle detector is an instrument that can record the passage of particles through it. From a teaching point of view, the *bubble chamber* is a particularly valuable detector because it provides a picture of the trajectories of charged particles travelling through it; the dark lines in figure 1 (more later) are examples.

The bubble chamber [1–4], invented by Donald A Glaser, consists of a tank of unstable transparent liquid—a superheated (roughly 2:1) mixture of neon and hydrogen in our case. When a **charged** particle forces its way through this liquid, the energy deposited initiates boiling along the trajectory—leaving a trail of tiny bubbles†. The superheated liquid is prepared by starting with the liquid held under pressure (about 5 atmospheres at a temperature of about 30 K for the Ne–H_2 mix) and then, just before the beam particles arrive, the pressure is reduced by suddenly expanding the volume by about 1% by means of a piston.

After the particles have passed through the liquid, the bubbles are allowed to expand until they are a few tenths of a millimetre across, big enough to be photographed by flash illumination. It is interesting to imagine the time scales involved: the relativistic particles cross the few metres of liquid in a few nanoseconds (1 ns = 10^{-9} s); the growth time is about a million times longer, $\sim$ 10 ms.

† The force responsible for the energy loss is the so-called Coulomb force (first published by Joseph Priestley, more renowned for his discovery of oxygen; see [5]).

Once the photographs are taken (more than one view is needed to reconstruct an interaction in 3D), the bubbles are collapsed by recompressing the liquid, and the bubble chamber is prepared for the next burst of beam particles.

The great advantage of bubble chambers is their ability to pick up details of complicated interactions—by following the trails of bubbles one can see subsequent interactions and decays of the products of the initial interaction.

Sadly, bubble chambers have recently become extinct. They could not cope with the huge event rates of current fixed-target experiments; nor could they be used with colliding beams. Nevertheless, like dinosaurs, they are remembered fondly! They served the particle physics community for 40 years. As experiments grew, needing millions of photographs to address current issues, larger groups of (about 10) collaborating laboratories emerged, paving the way for the huge collaborations of today's experiments—which typically involve about a thousand scientists from over a hundred laboratories around the world.

But, most of all, bubble chambers are remembered for their enduring images.

An exceptionally rich bubble chamber picture

Figure 1 shows part of a bubble chamber picture of an interaction between a high energy neutrino beam particle and a neon nucleus. The dark lines are the bubble trails; they are curved because the liquid is enclosed by a powerful magnet, the direction of the magnetic field being such that negative particles turn to the right. (Remember: neutral particles do not leave trails of bubbles because it is the electric force between the moving particle and the electrons of the atoms of the liquid that initiates bubble growth.)

Let us examine the picture closely.

- Several lines coming in from the bottom can be seen to be diverging; they are coming from the neutrino interaction, way upstream of the picture. (Look at the orientation of the lettering to determine top and bottom!)

- Some tracks are more curved than others. We shall now show that **the more curved the track,**

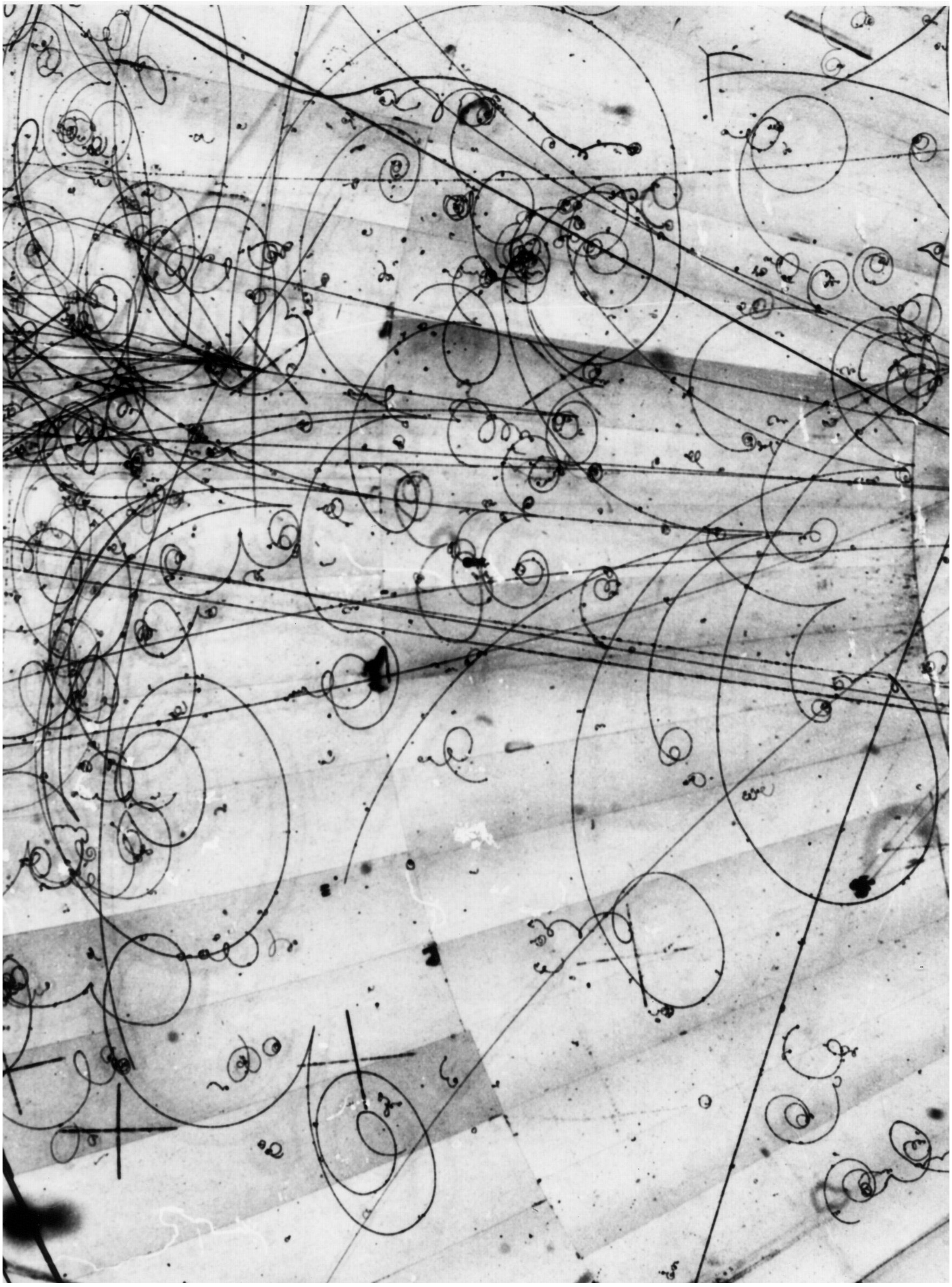
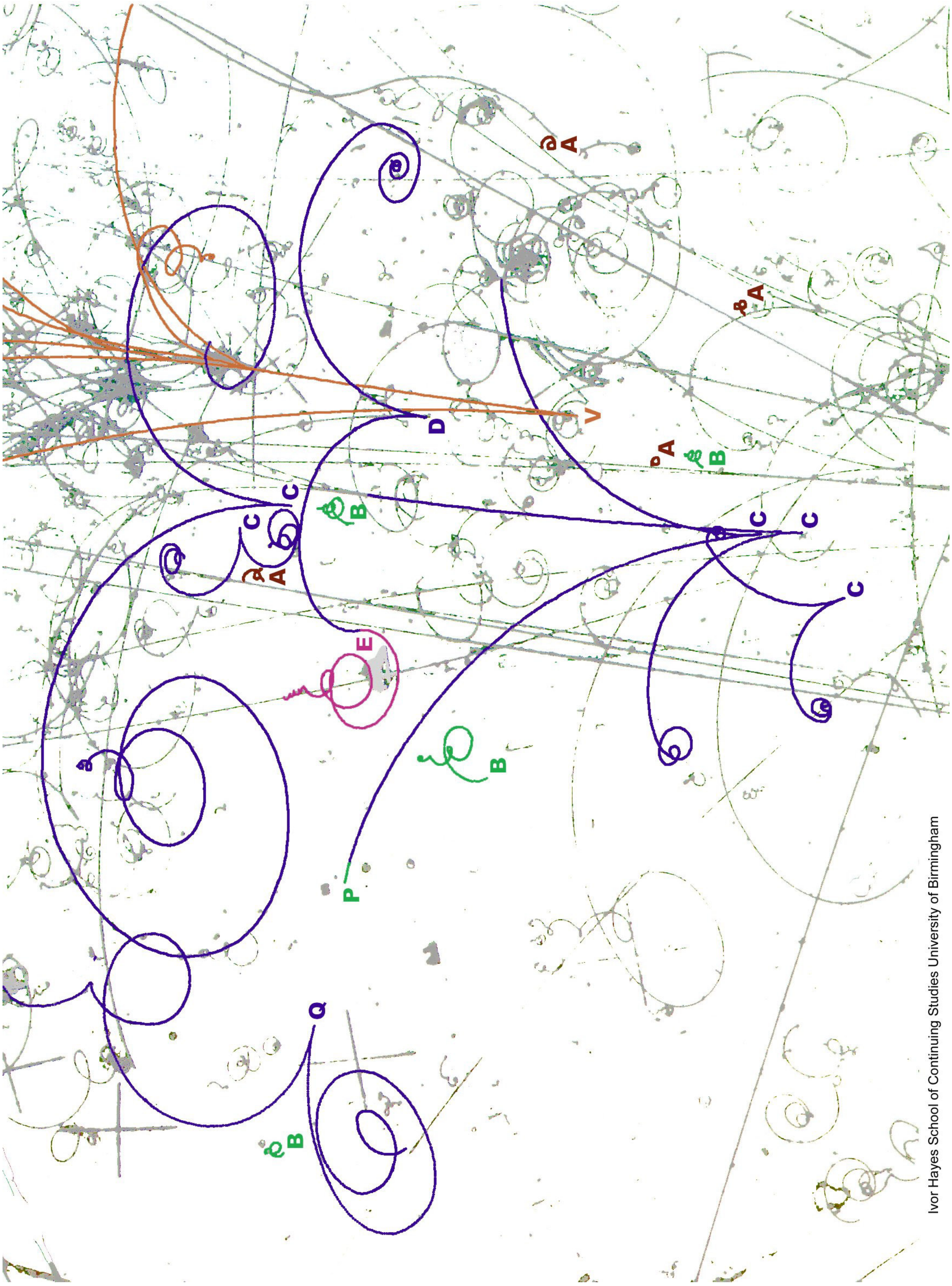


Figure 1. Part of a bubble chamber picture. The dark lines are trails of tiny bubbles created as charged particles force their way through a tank of transparent liquid enclosed in a powerful magnet. At P, the main point of interest, a positron (antielectron) in flight meets an electron and annihilation takes place. One of the photons from the annihilation *materializes* at Q. At E, a different positron seems to change



its sign. What has happened is that it has collided head-on with a stationary electron and has transferred all (within errors) its momentum to the electron. This shows that (within errors) the positron has the same mass as the electron. (This picture, from experiment E632 performed at the Fermilab 15 ft (4.6 m) bubble chamber, was found at the University of Birmingham. Graphics by Ivor Hayes.)

the lower the momentum of the particle that made it.

A particle of charge q travelling through a magnetic field B with a speed v experiences a force at right angles to its motion, given by Bqv if the motion is perpendicular to the field. This makes the particle follow a circular path of radius r, the motion being described by

$$Bqv = \frac{mv^2}{r}. \tag{1}$$

Rearranging this gives

$$p = (Bq)r. \tag{2}$$

This tells us that for a given field B and charge q, the momentum p is proportional to the radius of curvature r. This equation is valid in both non-relativistic and relativistic situations.

Here, nature has been kind: all charged particles that live long enough to travel a measurable distance have a charge equal or opposite to that on the electron e ($= -1.6 \times 10^{-19}$ coulomb).

- **Example 1.** In the Fermilab 15 ft (4.6 m) diameter bubble chamber, where this picture was taken, the magnetic field was 3 T (the Earth's magnetic field at the surface near the magnetic north pole is 6.2×10^{-5} T).
  Working in SI units, show that the momentum of a particle travelling with a radius of curvature of 1.11 m is 5.33×10^{-19} kg m s^{-1}.
- **Solution.** Using (2),

$$\begin{aligned} p &= 3\ \text{T} \times (1.6 \times 10^{-19})\ \text{C} \times 1.11\ \text{m} \\ &= 5.33 \times 10^{-19}\ \text{kg m s}^{-1}. \end{aligned}$$

[**Aside.** In the units used by particle physicists, this momentum is 1 GeV/c.]

• **Electrons spiralling due to 'bremsstrahlung'.** At several places, marked A, a low momentum negative track (it curls to the right) can be seen, beginning on a track of much higher momentum. These are electrons that have been 'struck' (remember, it is really the Coulomb force) by the passing charged particle, which is much more massive than the electron.

It is important to notice how these electron tracks spiral in, showing that they lose their energy at a considerable rate as they travel. This is in contrast to the massive particles that have struck them. This is due to the fact that, apart from losing energy by creating bubbles, electrons lose energy much more quickly by another process, known as *bremsstrahlung* (braking radiation).

This process, which is a consequence of the fact that all accelerated charges radiate, is important for electrons because they have small masses. One can argue as follows: for a particle of given charge, the amount of energy lost by bremsstrahlung depends on its acceleration; the acceleration is provided by the electric field due to the nuclei of the medium through which the particle is moving; by Newton's second law, the acceleration for a given force varies inversely with the mass. So, since the next lightest charged particle after the electron is the muon, which is over 200 times more massive, we do not expect much bremsstrahlung from particles other than electrons! (Especially since it is the square of the mass that counts!)

The upshot of all this is that an electron is instantly recognizable in our bubble chamber because its track will spiral. (In passing, this would not be true for a liquid hydrogen bubble chamber because the singly charged nuclei do not produce enough acceleration.)

• **The Compton Effect.** At several points, marked B, a lone (spiralling) electron can be seen. This is an electron that has been knocked out of an atom by a high energy photon, or γ-ray. The photon does not leave a track because it is electrically neutral. Such electrons are called *Compton electrons*. Can you find any more 'Comptons'?

• **Particles and antiparticles.** There are several points, marked C (two special ones, D and Q, will be discussed later), from which two spiralling tracks, one positive and one negative, are seen to emerge with zero opening angle. These are high energy photons *materializing*, in the field of a nucleus, into a positron–electron (e^+e^-) pair. (See [6] for detailed discussion.)

The first thing to note is that the positron tracks look similar to the electron tracks apart from curving in the opposite direction—they leave trails of bubbles, they spiral; there is nothing mystical about antimatter! An antiparticle has the opposite value of charge (and other additive, conserved quantities such as baryon number and lepton number) to that of the corresponding particle.

There are several more examples of *materializing* photons in the picture; can you find some?

• **'Weighing an antielectron'.** At the point marked E, the positron track that left D seems to change into a negative track of more-or-less the same curvature (momentum). What has happened is that the positron has made a head-on collision with an electron, transferring what looks like all its momentum to the electron—suggesting that the mass of the positron is equal to that of the electron. (Imagine collisions between snooker balls: in head-on collisions, one ball can transfer all its momentum to another. If, however, the balls were of different masses—think of a lead ball striking a polystyrene ball, or vice versa—one would not observe the complete transfer of momentum from one ball to another. See [7, 8] for further details.)

Many thousands of bubble chamber pictures were studied before this example was found.

• **A strange particle!** At the point marked V we see something that looks like a high energy photon materializing. However, following the (straighter) negative track, we see that it ends in an interaction from which a (dark) spray of other tracks emerges. Electron tracks do not interact in this way. Curvature measurements of the tracks leaving V show that the interacting track is a negative pion or antiproton from the decay of a neutral *strange* particle (a K^0 or $\bar{\Lambda}$) that was created in the original neutrino interaction upstream of our picture. Were it not for the existence of this high energy strange particle, this event would not have been analysed in detail, and the rare positron phenomena discussed in this paper would not have been found!

• **A positron annihilates in flight.** Finally we come to the **main feature to be discussed in this article.** At P, a positron annihilates in flight with an electron, and a photon that is produced materializes 11.5 cm away (in the bubble chamber) at Q. This is another classic but rare signal of a positron.

Looking at the annihilation closely, one gets the impression that all of the momentum of the positron (try to assess its curvature by eye) could have gone into the photon (look at the curvature of the tracks of the e^+ and e^- into which the photon materializes).

Is this actually the case? We will see that, within the limits of experimental error, the answer is 'Yes', a result that is, however, not consistent with relativity! The apparent conflict will then be resolved by showing that a second photon—of low enough energy to be accommodated by the errors in the energy measurements—must be created in the annihilation.

Details of measurement of bubble chamber tracks

In analysis of bubble chamber pictures, the momentum of a track is obtained by measuring, on at least two views so as to be able to reconstruct in three dimensions, the coordinates of several points along a track. Because of measurement errors, these point will not lie on a perfectly smooth curve. The curve which best *fits* through these points is then calculated, together with errors which give a feel for the spread of curves that could be considered consistent with the measured points. The radius of curvature (with error) of this curve gives (via equation (2)) the momentum of the track (with error).

In principle this is straightforward. In practice it is a complicated procedure. For one thing, the particles lose energy as they force their way through the bubble chamber liquid; so they are more curved at their ends than they are at their beginnings; this must be taken into account.

In the case of light particles like electrons, curvature changes due to bremsstrahlung are unpredictable and often quite severe, making a momentum measurement particularly difficult.

Using CERN's state-of-the-art fitting program, our momentum measurements for the e^+ approaching point P on the picture, and the γ coming from P and materializing at Q, are 200 ± 47 MeV/c and 265 ± 31 MeV/c respectively. (Systematic errors are negligible in comparison with these large statistical errors, forced upon us by the short lengths of track that can sensibly be used for measurement, typically 5–10 cm.)

We see that, within our measurement errors, the e^+ has given all its momentum to the photon that materializes about 11.5 cm along its line of flight.

Investigating whether this is really what has happened is the topic of the remainder of this article. First we gather a few basic relationships that describe the motion of particles moving with speeds close to that of light.

Some relativistic kinematics

In ordinary Newtonian mechanics, the kinetic energy of a particle is given by $p^2/2m$, obtained by replacing v in $mv^2/2$ by p/m.

In relativistic mechanics, the corresponding formula is

$$E^2 = p^2c^2 + m_0^2c^4. \tag{3}$$

We see that the expression for energy has two terms. For $p = 0$ (a stationary particle) $E = m_0c^2$: a stationary particle has 'rest' energy by virtue of its rest mass m_0.

For very high momenta, when $p^2c^2 \gg m_0^2c^4$, $E \approx pc$. (For photons, $E = pc$.)

The relation $p^2c^2 \gg m_0^2c^4$ holds for the particles discussed in this paper; they are 'highly relativistic' particles. For example: the positron approaching P has a momentum $\sim$ 200 MeV/c and a mass of 0.511 MeV/c^2; so $p^2c^2 \sim 40\,000$ MeV^2, while $m_0^2c^4 \sim 0.25$ MeV^2.

[**Aside.** Although a 200 MeV positron is highly relativistic, moving with a speed v of 99.999 67% the speed of light, its energy E is roughly 500 times smaller than that of the 100 GeV electrons and positrons made to collide head-on in large *storage ring* experiments such as LEP at CERN. These LEP electrons move at 99.999 999 998 7% the speed of light.

- **Problem.** Verify the above speeds.
  Hint. Square $E = \frac{1}{\sqrt{1-(v/c)^2}}m_0c^2$, which is the relativistic version of $E = \frac{1}{2}mv^2$; replace $[1 - (v/c)^2]$ by $(1 + v/c)(1 - v/c) \approx 2(1 - v/c)$ for $v \approx c$, and rearrange to give

  $$1 - \frac{v}{c} \approx \frac{1}{2}\frac{(m_0c^2)^2}{E^2}.$$

  Use 0.511 MeV for m_0c^2 for the electron. It comes out in a couple of lines.

For an introduction to the Large Electron Positron collider LEP, see the website:

http://www.cern.ch/Public/ACCELERATORS/LepAcc.html.

Did you know that the World Wide Web was invented at CERN to help particle physicists communicate with each other?!]

- Another useful formula for the energy of a relativistic particle relates the kinetic energy T to the total energy E:

$$E = T + m_0c^2. \tag{4}$$

- **Example 2.** Show that for a highly relativistic ($T \gg m_0c^2$) particle

$$pc \approx T + m_0c^2. \tag{5}$$

Solution. Equating expressions for E^2 from equations (3) and (4) yields

$$p^2c^2 + m_0^2c^4 = (T + m_0c^2)^2. \tag{6}$$

Subtracting $m_0^2c^4$ from both sides, using the binomial theorem $[(1+x)^{-1/2} \approx 1 - x/2$ for small $x]$, and taking square roots gives

$$pc = \sqrt{T^2 + 2m_0c^2T} = T\sqrt{1 + \frac{2m_0c^2}{T}} \approx T(1 + m_0c^2/T) = T + m_0c^2. \tag{7}$$

Positron–electron annihilation to one photon is impossible

This question is most simply addressed by considering the possibility of producing, in a head-on collision between an e^- and an e^+ having equal but opposite momentum, a single photon.

Applying energy and momentum conservation, one can see that the photon would have an energy equal to the total energy of the incoming e^-e^+ pair and a momentum of zero. This is **impossible** because a photon's energy and momentum are related by $E = pc$.

[**Aside.** It is instructive for those new to relativistic calculations to see how the same problem, addressed in the rest frame of the e^-, say, yields the same answer, albeit in a physically less transparent manner.

Consider a high energy positron of energy E and momentum $\boldsymbol{p}$ approaching a stationary† electron of (rest) mass m (figure 2). Let the proposed single photon γ in the final state have energy E' and momentum $\boldsymbol{p}'$.

Figure 2 looks 'wrong' because we know almost instinctively that momentum conservation says that the photon's momentum must be along the direction of motion of the incoming e^+, and equal in magnitude to the momentum of the e^+, i.e. $p' = p$.

† Compared with the speed of the highly relativistic incoming positron, an electron in the bubble chamber can be taken to be at rest.

Figure 2. A positron e^+ of energy E, momentum $\boldsymbol{p}$ and mass m approaches a stationary electron e^- also of mass m. It is hypothesised that, after their annihilation, a single photon γ of energy E' and momentum $\boldsymbol{p}'$ is created.

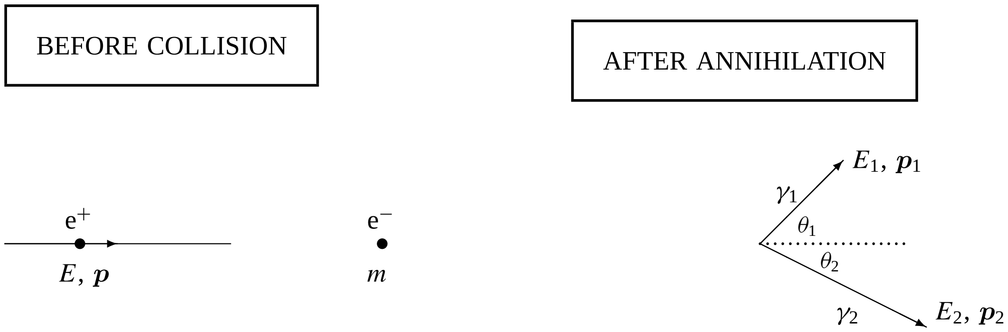


Figure 3. A positron e^+ of energy E, momentum $\boldsymbol{p}$ and mass m approaches a stationary electron e^- also of mass m. After their annihilation, two photons γ_i, $i = 1, 2$, are created with energies E_i, momenta $\boldsymbol{p}_i$ and directions θ_i with respect to that of the incident e^+.

Energy conservation (relativistic) then gives

$$(p^2c^2 + m^2c^4)^{1/2} + m^2c^4 = pc. \qquad (8)$$

(Remember: positron and electron masses are equal.)

Since m is positive, the left-hand side of this equation is clearly greater than pc, the value of the right-hand side. So, there is no value of momentum that satisfies this equation, because we have considered an impossible process: e^+e^- annihilation cannot yield just one photon.

The physics is the same in both frames of reference!]

Usually, e^+e^- annihilation yields two photons, which we shall refer to as γ_1 and γ_2 from now on. (γ, pronounced 'gamma', is the third letter of the Greek alphabet.)

Now we need to reconsider our bubble chamber picture which, within measurement errors, shows an annihilation in which the photon takes up all the energy of the incoming e^+.

Kinematics of e^+e^- annihilation in flight

Let us ask a specific question: **what is the maximum fraction of the incoming positron's energy that can be taken up by the more energetic of the two photons?** (The photograph suggests it could be 100%.)

Consider an e^+ of energy E and momentum $\boldsymbol{p}$ approaching a stationary electron of mass m (0.511 MeV/c^2) from the left, producing two photons with energies E_1 and E_2, and momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ (figure 3).

[**Aside.** Before reading on, the student should spend a few moments looking at figure 3 and trying to imagine what is going on, concentrating on how γ_1, say, could get a maximal share of the energy.

We will argue firstly that the best final state for sharing the energy most unequally involves **no motion perpendicular to that of the incoming e^+**—because any **such** motion must involve γ_2 having as much momentum as γ_1 (conservation of momentum at right angles to the e^+ line of flight), thus increasing γ_2's overall share.

Next we need to persuade ourselves that this means that γ_1 moves to the right in the laboratory, and γ_2 to the left; they cannot both be moving to the right. We can see this by considering the process in the centre-of-mass system, where the right-moving e^+ meets the left-moving e^-, which has an equal and opposite momentum. As seen from the laboratory frame, this centre-of-mass frame moves to the right (with a lower speed than that of the e^+).

By momentum conservation in the centre-of-mass frame (sometimes known as the centre-of-momentum frame), γ_1 and γ_2 must emerge back-to-back with equal and opposite momentum.

Viewed from the laboratory, they will also be back-to-back because, in moving from the CM frame to the laboratory frame (boosting to the left), we cannot catch up with the left-moving photon! (They will not have equal and opposite momenta in the laboratory.)

In summary: for a γ to have the maximum (minimum) possible energy it must move to the right (left) with $\theta = 0^\circ$ (180°).

We now proceed to calculate these maximum and minimum energies.]

From momentum conservation

$$\boldsymbol{p}_2 = \boldsymbol{p} - \boldsymbol{p}_1. \tag{9}$$

Energy conservation gives

$$E_2 = E + mc^2 - E_1. \tag{10}$$

Squaring (9),

$$p_2^2 = p^2 + p_1^2 \mp 2pp_1. \tag{11}$$

The minus (plus) sign is for θ being 0° (180°), which corresponds to maximum (minimum) γ energy.

Multiplying (11) by c^2, using $E_1 = p_1c$ and $E_2 = p_2c$ for the photons, and $E^2 = p^2c^2 + m^2c^4$ for the positron, we get

$$E_2^2 = E^2 - m^2c^4 + E_1^2 \mp 2pcE_1.$$

Squaring (10) gives

$$E_2^2 = E^2 + m^2c^4 + E_1^2 + 2Emc^2 - 2E_1mc^2 - 2EE_1.$$

Equating values of E_2^2 in the last two equations, subtracting $(E^2 + E_1^2)$ from both sides, and bringing terms containing E_1 to the left-hand side, we have

$$E_1(E + mc^2 \mp pc) = mc^2(E + mc^2).$$

Hence,

$$E_1 = \frac{mc^2(E + mc^2)}{E + mc^2 \mp pc}. \tag{12}$$

[**Example 3 (for advanced students).** Beginning with figure 3, derive equation (12) by considering (a) energy conservation and (b) momentum conservation (there are components along, and perpendicular to, the line of flight of the e^+).

(This method is mathematically more elegant, but provides less of the 'feel' that an experimentalist might prefer. Best is both!)

Solution. See Appendix.]

We are now in a position to estimate the maximum and minimum photon energies for our annihilation of a highly relativistic positron in flight. Using $E = T + mc^2$ (equation (4)) we can re-express (12) in terms of the kinetic energy T:

$$E_1 = \frac{mc^2(T + 2mc^2)}{T + 2mc^2 \mp \sqrt{2mc^2T + T^2}}. \tag{13}$$

This simplifies surprisingly nicely! Dividing numerator and denominator by $(2mc^2 + T)$ gives

$$E_1 = \frac{mc^2}{\left(1 \mp \frac{T\sqrt{1+2mc^2/T}}{T(1+2mc^2/T)}\right)} = \frac{mc^2}{1 \mp (1 + 2mc^2/T)^{-1/2}}. \tag{14}$$

For $T \gg mc^2$ the binomial theorem $[(1+x)^{-1/2} \approx 1 - x/2$ for small $x]$ gives

$$E_1 \approx \frac{mc^2}{1 \mp (1 - mc^2/T)}$$
$$= T \quad \text{for minus sign (maximum)}$$
$$= mc^2/2 \quad \text{for plus sign (minimum).} \tag{15}$$

So: for the annihilation in flight of a highly relativistic e^+, the maximum energy that one γ can take is the whole of the kinetic energy of the e^+, leaving a mere half of an electron's rest energy (≈ 0.26 MeV) for the other photon.

For the 200 MeV positron in the photograph this means that, within measurement errors, all the e^+'s energy can go into one photon, with the other photon having such a low energy that it would not leave a track in the bubble chamber.

Summary

An attempt has been made to put on record a detailed, stand-alone discussion of a quite extraordinary bubble chamber picture, containing:

- an unusual interaction—the annihilation of a high energy positron in flight;
- evidence of several other exotic phenomena.

It is hoped that the information provided can be of value to

- school teachers of more traditional courses, by providing qualitative illustrations of basic concepts such as conservation laws and the emission of electromagnetic radiation by accelerating charges;
- university teachers, by providing new illustrative examples with a full treatment of the relativistic kinematics;
- good students wishing to read beyond their syllabus.

Acknowledgments

It is a pleasure to acknowledge the work of the Fermilab bubble chamber crew, and the film scanning and measuring staff of the laboratories participating in experiment E632. I would especially like to thank Peter Faulkner at Birmingham for his alertness in spotting the rare phenomena on the picture discussed in this article.

Appendix

Before tackling example 3, we introduce a labour-saving device—what practising physicists call the '$c = 1$ convention'.

Exercise. While following the kinematics above, you may have felt that the cs are a bit of a nuisance! You may also have noticed that ms are always multiplied by a c^2 while ps are always multiplied by a c. So, why not drop all cs from the beginning and put them in at the end?!

Derive equation (12) without cs and then verify, by putting them back in, that everything has worked out consistently.

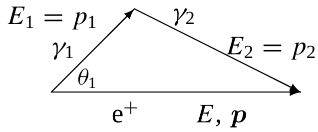


Figure 4. A momentum-vector triangle showing the energies E_i and momenta $\boldsymbol{p}_i$ of the two photons γ_i, $i = 1, 2$, emerging from the annihilation of a positron e^+ of energy E and momentum $\boldsymbol{p}$ with a stationary electron (not shown). The angle θ_1 gives the direction of γ_1 with respect to $\boldsymbol{p}$.

Solution to example 3

Using the '$c = 1$ convention', energy conservation gives

$$E + m = E_1 + E_2. \qquad (16)$$

Since we are going to equate values of E_2^2 as before, this can be rewritten as

$$E_2^2 = (E + m - E_1)^2. \qquad (17)$$

Next we draw a vector diagram representing $\boldsymbol{p} = \boldsymbol{p}_1 + \boldsymbol{p}_2$ (figure 4).

Notice that, in the '$c = 1$ convention', the photon relationship '$E = pc$' has become '$E = p$', where p is the magnitude of the 3-momentum.

Using the cosine rule for the vector triangle

$$\begin{aligned} E_2^2 &= E_1^2 + p^2 - 2pE_1 \cos\theta_1 \\ &= E_1^2 + E^2 - m^2 - 2pE_1 \cos\theta_1. \qquad (18) \end{aligned}$$

Equating values of E_2^2 from equations (17) and (18), and subtracting $E^2 + E_1^2$ from both sides, yields

$$m^2 + 2mE - 2EE_1 - 2mE_1 = -m^2 - 2pE_1 \cos\theta_1 \qquad (19)$$

Rearranging and solving for E_1:

$$E_1 = \frac{m(E + m)}{E + m - p\cos\theta_1}. \qquad (20)$$

The largest and smallest values of E_1 correspond to $\cos\theta_1 = 1$ and -1, which occur for $\theta_1 = 0^\circ$ and 180° respectively.

Notice now that, if we multiply ms by c^2 and ps by c, we get equation (12), as set.

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