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The uncertainty principle, virtual particles and real forces

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Abstract

This article provides a simple practical introduction to wave–particle duality, including the energy–time version of the Heisenberg Uncertainty Principle. It has been successful in leading students to an intuitive appreciation of *virtual particles* and the role they play in describing the way ordinary particles, like electrons and protons, exert forces on each other.

This resource article is based on experience of teaching this topic at an introductory level to a variety of audiences. Several approaches have been tried; this has been the most successful because the crucial argument—that the better the frequency of a wave is to be measured, the more time is needed—is invariably produced by class members, **and seems intuitively reasonable to them**.

The material is divided into several sections:

(1) Comments on preliminary ideas from classical physics.
(2) A practical introduction to $E = hf$ and $p = h/\lambda$ (wave–particle duality).
(3) The energy–time uncertainty relation as a consequence of (2).
(4) An argument to motivate considering the possibility of processes that violate energy conservation, based on the energy–time uncertainty relation (the ‘magic loophole’)
(5) Using this *loophole* to speculate about such processes, eventually arriving at the modern way of describing a force between two real particles in terms of the passage of *virtual particles* from one to the other.
(6) Emphasizing the power of what has been done by applying this picture to some real forces.
(7) Drawing attention to the dangers of taking an oversimplified approach too literally.

Classical physics input

- **Kinetic energy and momentum.** It will be assumed in this paper that the concepts of kinetic energy E and momentum p have been discussed. (It takes typically a two-hour session to get non-scientists to feel comfortable with these ideas.)
  For the more mathematically inclined, it is worth pointing out that the three equations
  $$E = \tfrac{1}{2}mv^2 \qquad p = mv \qquad E = \frac{p^2}{2m}$$
  show that if we know any two of the quantities E, p and v (speed) for an object, we can calculate its mass m. (In passing, the same is also true for relativistic particles [1].)
- **Light is a wave.** It is also assumed that Young’s two-slit experiment demonstrating that light is a wave has been performed. Non-scientists are generally delighted to be able to measure something as small as the wavelength of light (less than one thousandth of a millimetre) by measuring just two quantities—the distance from a He–Ne laser to a screen and the separation of two regions of high intensity—and then using the method of similar triangles! (To get here from scratch with non-scientists takes 1–2 hours.)

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Wave–particle duality: a practical approach

Assuming that light is a wave and that electrons are particles, one needs two experiments to establish wave–particle duality. Two of the simplest are the following:

(1) **The photoelectric effect.** When light of frequency f is shone onto a metal surface, electrons can be knocked out. Increasing the intensity of the light does not affect the energy with which individual electrons emerge, a fact that cannot be described in terms of classical physics, which would predict electrons of greater energy. Einstein was awarded his Nobel prize *for his contributions to mathematical physics, and especially for his discovery of the law of the photoelectric effect.*

Einstein's picture is the following: the light striking the metal surface should be thought of as a stream of particles called *photons,* each having a kinetic energy $E = hf$, where h is a constant of nature known as Planck's constant. The more intense the light, the greater the **number** of photons striking the surface per second and the **number** of electrons ejected. The energy with which they are ejected does not vary with intensity because photons of a given colour have the same energy whether they are in a weak beam or an intense one. (The chance of more than one photon striking the same electron is ignored.)

(2) **The two-slit experiment with electrons.** In 1974, Jönsson [2], overcoming considerable technical difficulties, succeeded in firing a beam of electrons at two slits and observing the interference pattern. This experiment is clearly described with many helpful illustrations in an excellent book called *The Quantum Universe* by Hey and Walters [3].

From the separation of the maxima of the interference pattern, the wavelength λ of the electron wave can be determined by Young's method.

By varying the momentum of the electron beam and measuring the corresponding wavelength, one can show that $p\lambda = h$; Planck's constant again! This relationship tells us that the higher the momentum p of a particle, the smaller is its de Broglie wavelength.

Perhaps it is also worth pointing out that here is an experiment that is simple to describe and gives a method of determining Planck's constant.

The equation $p = h/\lambda$ is known as the de Broglie relation after the Frenchman who predicted wave–particle duality.

An aside: Why do particle physicists need high-energy accelerators?

Particle physicists are microscopists—they study the structure of neutrons and protons. It is a rule of microscopy that you cannot see anything smaller than the wavelength of the radiation you are shining on it; one way to come to terms with this idea is by means of the following analogy: you cannot sense the details of Braille with a blackboard eraser, but you can with the point of a pencil.

To get the very small wavelengths needed to study the interiors of neutrons and protons, particle physicists use particle beams of very high momentum ($\lambda = h/p$). So, a particle physics laboratory like CERN is a huge microscope! □

We now have the two basic formulae of quantum mechanics: $E = hf$ and $p = h/\lambda$, relating the particle properties E and p to the wave properties f and λ.

The waves concerned are, however, mysterious ones: they determine the relative probabilities of where an electron (that has gone through a two-slit experiment in our case) will be found. We do not understand these waves in the way we understand other waves such as sound waves, and it is amazing to remember this when one thinks of the power that quantum mechanics has given us. A good discussion of the mystery of quantum mechanics is to be found in *The Quantum: Illusion or Reality* by A Rae [5].

Here we take a different approach and try to come to terms with these quantum waves (without mathematics[1]).

The approach to be taken in the next section to show how the Heisenberg Uncertainty Principle

[1] By *without mathematics* we mean *for those who claim not to be able to do mathematics,* typically graduates in non-scientific disciplines who, although they may have distant school memories of mathematics, seem to have lost confidence in their abilities. Such people are very often capable of following a mathematical argument, provided every step is performed in detail, and appreciate the experience. People are very happy to attend classes on the appreciation of music or poetry without expecting to be able to play an instrument or write a poem afterwards. Why should the same not be true of mathematics, which shares aesthetic qualities with both?!

Worked example for students

Problem. Derive an expression for the momentum p of an electron of mass m and charge e that has been accelerated through a voltage V.

In his experiment, Jönsson accelerated electrons through 50 kV. What de Broglie wavelength would these electrons have? (Ignore relativistic effects that are beginning to become significant at such voltages; see [4].)

Solution. A stone of mass m dropped from a height h_2 to a height h_1 loses a potential energy of $m \times$ (the gravitational potential difference) = $m \times (gh_2 - gh_1)$.

Likewise, an electron of charge e *dropped* through a potential difference of V volts loses a potential energy $e \times V$. Here we come across a little problem with units: an electron accelerated through 1 volt acquires an energy of 1 *electron-volt* (eV) and 1 eV = 1.602×10^{-19} J.

The potential energy lost appears as kinetic energy gained. Thus:

$$\frac{p^2}{2m} = eV \quad \Longrightarrow \quad p = \sqrt{2meV}.$$

The de Broglie wavelength is then given by

$$\lambda = \frac{h}{\sqrt{2meV}}.$$

The relevant numbers: $h = 6.63 \times 10^{-34}$ J s, $e = 1.602 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg. Using these and the conversion factor from electron-volts to joules:

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times (50 \times 10^3) \times 1.602 \times 10^{-19}}}.$$

These give $\lambda = 5.49 \times 10^{-12}$ m, which is much smaller than the diameter of an atom, about $\sim 10^{-10}$ m.

is a consequence of wave–particle duality is the following: **although we do not understand these quantum waves as we would like, we will assume that, whatever properties other waves share, the quantum waves have them too.**

An aside

As an example, let us repeat the argument often given for **why atoms have energy levels**. We know that a wave confined to a guitar string vibrates with a set of allowed frequencies—the fundamental frequency and higher harmonics. The same is true of all confined classical waves. If we assume that the motion of an electron in a hydrogen atom is influenced by wave properties, as the motion of a free one is seen to be in the two-slit experiment, then we could describe the electron in a hydrogen atom as a confined electron wave. Now, confined waves have definite frequencies, so we can say that an electron in a hydrogen atom has definite frequencies: $f_1, f_2, f_3, \ldots$ etc. It is no more than a convention (based on the knowledge from $E = hf$ that h times a frequency has units of energy) to say that the electron (or electron wave in the H-atom) has a set of allowed values of hf: $hf_1, hf_2, hf_3, \ldots$ etc. We can call these *energy levels.*

So, atoms have energy levels because they are confined electron waves! □

The Heisenberg Uncertainty Principle

The main merit of the argument to be presented here is that it has proved successful in getting non-scientists to the point where they think they have an intuitive feel for the following statement: **to**

measure the energy of a system with accuracy[2] **ΔE one needs a time longer than $rh/\Delta E$, where r is a positive number.**

It is felt that, since this statement of the Heisenberg Uncertainty Principle is the key to appreciating many exceedingly exciting phenomena[3] in an intuitive way, one should not worry too much in the first instance if one has not been rigorous.

Some of these pinnacles of 20th century culture should not be denied to non-scientists just because they do not know much about waves! It must be pointed out, however, that many of these non-scientists (and scientists from other disciplines) are not at all satisfied with just seeing the peaks, as one might from a plane just see the Himalayas poking out of a blanket of cloud; they want to be taken up; they want to get a feel for the climb.

The starting point for the argument is that we assume that our quantum waves share all properties shared by ordinary waves. Here we start with sound. The teacher/lecturer is invited to sing two nearly identical notes in **very** short bursts. The students are asked to say which is the higher. (The same can be done with *scientific* sound generators but the impact on the students is not the same! The fact that the teacher is making a spectacle of himself usually galvanizes the students into trying hard to put him out of his misery, by thinking!)

Usually the students say they cannot tell which note is higher. The teacher can then try again, holding the notes a little longer. The students are now able to distinguish the notes and, on being asked why they could do it the second time, respond that the short note doesn't sound musical, that one can determine a note (measure frequency f with a small uncertainty Δf) better the longer it is played.

(It is vital at this point to make sure that everyone agrees that the previous statement is intuitively reasonable. Typically, about 80% are happy with this. The remaining 20% are usually the ones who seem to know more and want the argument tightened up. To try to do so at this stage can destroy the confidence of the 80%!)

Another way of saying this is that if we specify a precison Δf we want for a frequency measurement, we need a time greater than a certain amount that depends on Δf. A little discussion about fractions for the non-scientists makes the following re-statement of their own earlier observation acceptable:

$$\text{Measurement time} \geqslant r/\Delta f$$

where r is a positive number.

This is the crucial statement about waves that we need. Let us assume that the same relationship is true for all waves, in particular the mysterious quantum waves. For these we know that $E = hf$. Multiplying the top and bottom of the right-hand side of the above equation by h we get

$$\text{Measurement time} \geqslant rh/h\Delta f = rh/\Delta E$$

(since h is a constant).

This is the Heisenberg Uncertainty Principle in the form we want it. It states that if we want to measure the energy of a quantum system (something we want to discuss using quantum mechanics—an electron, for example) with accuracy ΔE we need a time greater than $rh/\Delta E$.

At this stage one can point out that if we had treated the discoveries of wave–particle duality more mathematically, we would have been able to derive this relationship (using the same physics: $E = hf$ and $p = h/\lambda$) and find that $r = 1/4\pi$.

'The magic loophole'

We will now see that a consequence of the Heisenberg Uncertainty Principle is that we can take seriously the possibility of the existence of energy non-conserving processes—provided the amount by which energy is not conserved, $E_{\text{violation}}$, exists for a time **less than** $h/4\pi E_{\text{violation}}$. This idea will then form the basis of a discussion of the *Exchange Model of Forces.*

Because the step from classical physics to quantum physics is at least as great a change of world view as was the change to classical physics from the Greek world view, let us adopt the dialogue style of Galileo! (You might even

[2] For non-scientists it is necessary to spend a little while on this concept of *accuracy*. An example: try to arrange to eat in one day a diet corresponding to an energy E of 2000 Calories (8.4 kJ). Variation in the energy content of slices of bread, say, will introduce an *error* or *uncertainty* of 50 Calories. Here we say that the energy E was measured with an *accuracy* $\Delta E = 50$ Calories.

[3] α decay, the modern view of forces to be discussed here, radiation from tiny black holes, etc.

be able to peruade two class members to play the parts!)

Master I propose that it is not impossible to have a process in which a state with energy E_{B} for Before, becomes a state with a **different** energy E_{A} for After.

Pupil Surely, you're joking, Master! Even I know that energy is conserved (if one remembers that relativity tells us that mc^2 is a form of energy).

Master Tell me how you would set about convincing me that I am wrong.

Pupil Well, may I assume that you still accept what you taught me, that there is only one test of a truth in science, and that is experiment?

Master Yes.

Pupil Then I would suggest that we could resolve our differences by taking your process and measuring the energy before, E_{B}, and the energy after, E_{A}, and seeing if they were the same.

Master Very well. Consider the following picture:

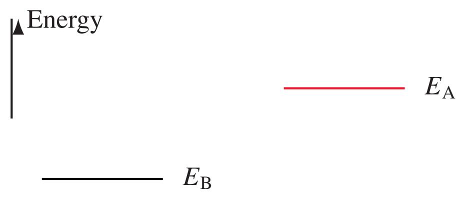

Figure 1.

How well do you have to measure the energies?

Pupil Well, clearly, if the error on either E_{B} or E_{A} were bigger than the gap $E_{\mathrm{A}} - E_{\mathrm{B}}$ between them, we would not be able to tell whether they were different or not. Maybe I could put it another way: if the net error (whatever that means!) on the two measurements is bigger than $E_{\mathrm{A}} - E_{\mathrm{B}}$, I could not tell that a change had taken place.

I guess what I am saying is that, give or take a factor of two that one might want for safety, the measurement accuracy must be less than the gap. So, in principle, given good enough apparatus, I could always tell if E_{A} were different from E_{B}.

Master Wait a moment! You did well to remember that mc^2 is a form of energy; you have one foot, at least, in the 20th century! But you have not taken wave–particle duality into account. Have we not seen that it takes time to make a measurement in the atomic and subatomic worlds? Tell me, if I were thinking of something from this realm of nature, how long would it take you to make your measurement with accuracy $E_{\mathrm{A}} - E_{\mathrm{B}}$?

Pupil Well, according to the Heisenberg Uncertainty Principle, I would need a time of **at least** $h/4\pi(E_{\mathrm{A}} - E_{\mathrm{B}})$.

Master Quite correct! But now I have you!

As you know, we are now living in strange times. Our bosses are being paid to play management-studies games with us; these include moving goalposts so that we have to move on to something new before we have time to finish what we are doing and complain that it is a waste of time! I hope I will be forgiven for using the same trick for something for which it is not really intended—doing something useful!

As you just pointed out, to make your measurement of $E_{\mathrm{A}} - E_{\mathrm{B}}$ with the desired accuracy, you would need a time of **at least** $h/4\pi(E_{\mathrm{A}} - E_{\mathrm{B}})$. Now, I'll move the goalposts! Consider the following, slightly modified, sequence of states:

(1) An initial state, as before, with energy E_{B}. Let it exist for a long time so that you can measure its energy very well.
(2) A state of energy E_{A} that exists for a time **less than** $h/4\pi(E_{\mathrm{A}} - E_{\mathrm{B}})$.
(3) A final state of the same energy E_{B} as the initial state. Let this state exist for a long time too, so that you can measure its energy very well.

Let me draw another picture to concentrate your mind:

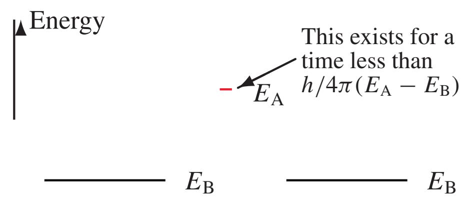

Figure 2.

Tell me: how could you show that this sequence of processes could not occur?

Pupil [Long pause ... the reader is invited to try to answer the question.]

I see that I have a problem! I do not have the time to measure the energy E_{A} with sufficient accuracy to tell whether it is different from E_B or not. So I cannot say that such a sequence is impossible. You have through quantum mechanics found a loophole! (You should have been an accountant!)

Master That's right. But, we must be careful. I am not claiming that such sequences of processes do occur in nature; I am just saying that they are not, in principle, forbidden.

Pupil What you have explained is fascinating from a philosophical point of view, but is it of any value as far as science is concerned? Surely, since, as we have just seen, it is impossible to demonstrate the existence of these energy-violating processes because they do not exist long enough to be measured with sufficient accuracy, they cannot be of value to a discipline that is based on experiment?

Master Amazingly, what you say is not quite true[4]. Using our loophole we can provide a valuable insight into the way two ordinary particles such as electrons exert forces on each other. One can go even further and use the same ideas to describe important properties of nuclear forces.

Pupil Please, before you do that, can you give me an example to think about? Your figure 2 is completely abstract. Can you give me an example of the three consecutive states of energies E_B, E_A and E_B respectively?

Master Yes.

(1) A stationary electron with energy $E_B = m_e c^2$, where m_e is the electron's rest mass.
(2) Let the electron emit a photon γ of energy E_γ and recoil with a momentum equal and opposite to that of the photon (we must conserve momentum, we only have a loophole for energy!). This state has energy $E_A = m_e c^2 + E_\gamma + \text{KE(electron)}$, where KE(electron) is the kinetic energy of the recoiling electron. Here

$$E_{\text{violation}} = E_\gamma + \text{KE(electron)}.$$

(3) Let the final state be the same as the initial state, and come into existence in a time less than $E_{\text{violation}}$.

[4] See *The Feynman Lectures on Physics* Vol. I, 38-6, for a refutation of the assertion that 'unless a thing can be defined by experiment, it has no place in a theory'.

We know that such a sequence is impossible from the point of view of classical physics—but it only violates energy conservation for a time governed by the Heisenberg Uncertainty Principle.

Pupil Yes, but...

Master No! Please do not, at this point, ask *how does the photon know how to come back?* or any such (perfectly reasonable) question. I have discovered the loophole and am trying to see if I can use it to help me imagine a physical process—remember, the loophole is a consequence of wave–particle duality, which is having considerable success in describing phenomena that are absolutely impossible to contemplate in terms of classical physics—like the existence of energy levels in atoms.

When I first got to this point, I had the feeling that thinking like this might give some insight. I was not expecting much more, but I was very wrong. If you bear with me for five minutes, I will try to convince you of the following:

- It is possible to picture the repulsion of two electrons in terms of the passage of photons from one to the other; the intermediate state, consisting of two electrons and a photon, is one of our energy-violating states that exists for such a short time that it is not possible to make a measurement to show that it existed. (By the way, perhaps I should have commented that photons are the 'natural' carriers for electromagnetic forces because we know from Maxwell that accelerated charges radiate electromagnetic radiation.)
- Short range nuclear forces can be pictured in the same way.

The crucial difference is that in the nuclear case the force-carrying particle is not a photon, which has zero rest mass. Conversely, the fact that the electrical force of repulsion extends to infinity is a consequence of the fact that the photon has no rest mass.

Virtual particles

Master Let us return to our example of the electron emitting a photon and recombining with it within the time limit imposed by the Uncertainty Principle—we call the intermediate state a *virtual state* composed of a *virtual photon* and a *virtual electron.*

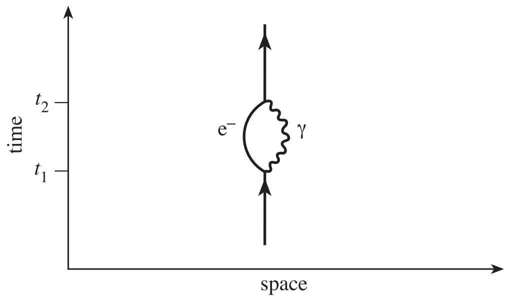

Figure 3.

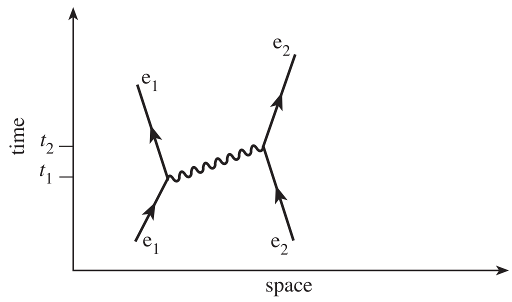

Figure 4.

Feynman has introduced a way of imagining such processes in terms of pictures now known as *Feynman diagrams*—two-dimensional space-time graphs. Our example of a stationary electron emitting a photon and recombining with it would be pictured as in figure 3.

The vertical parts of the graph correspond to the electron remaining at the same point—the classical view of a stationary particle. (If the electron were moving with a steady speed, these lines would be moving at an angle to the vertical.)

The loop in the middle corresponds to the virtual state. One should not try to take this picture too literally and try to give a running commentary of the evolution of the intermediate state. The picture just tells us that we have imagined the two-stage process I gave you as an example.

The Exchange Model of Forces

Master Now we are set up to present the quantum picture of forces.

Let us consider two electrons approaching each other with steady speed. They may or may not be heading straight towards each other. On our schematic Feynman diagrams we only have one space dimension and so both the above possibilities would look similar!

At some moment t_1 let the electron on the left of figure 4 emit a *virtual photon*. According to the Heisenberg Uncertainty Principle this virtual state must revert to one of the same energy as the original one within a time less than $E_{\text{violation}}$. We have already discussed one possibility in figure 3.

Now, in figure 4, we have a different possible outcome. If electron e_2 comes into the vicinity of the virtual photon before it is *due back*, it can *absorb* the virtual photon. Provided this happens in such a way that **the total energy of the two electrons before and after the intermediate state is the same**, we would not have violated any rule of physics. So, here is a second example of a process of the kind being advocated.

Let us spell it out once again, looking at the Feynman diagram in figure 4. There are several stages:

(a) Before t_1 two electrons approach each other just like classical particles.

(b) At t_1, electron e_1 emits a virtual photon to the right, let us say. In which case, by momentum conservation, e_1 will recoil to the left.

(c) At some time t_2, less than $h/4\pi\,E_{\text{violation}}$ later, electron e_2 absorbs the photon in such a way that two things happen:

- The total energy of the two electrons is equal to what it was before the intermediate virtual state.
- Electron e_2 recoils to the right because of the momentum it picks up from the photon.

(d) The two electrons move on like classical particles.

If we look at figure 4, it looks as if the particles have repelled each other! From wave–particle duality and some speculative thinking we have arrived at a picture or model of how one electron can exert a force on another.

This is our first glimpse of the modern way of looking at forces—the so-called *Exchange Model of Forces*, in which forces are described in terms of the *exchange* of virtual particles.

When I first saw this, I got very excited because I realized that this was a way of thinking about forces without having to worry about Faraday's weird 'lines of force'!

At this point, hands shoot up and all sorts of questions are asked and objections raised. This is a relief, because it means that the questioners have a reasonable grasp of the arguments that have been presented. In the next section some of these questions will be discussed.

First questions on the 'Exchange Model of Forces'

• **Question** (asked by teachers). The use of the word *force* here can cause difficulties—because children are taught that forces produce accelerations or change shapes. Would it be appropriate to use the word 'interaction' here?

Answer Yes; it is quite common to use the special word *interaction* for force at the most fundamental level—the exchange of a virtual particle.

(Don't forget that decays are also interactions: a hydrogen atom decaying from an excited state to a lower state, for example, would look like the left half of figure 4; and there would be no virtual particles because there is no need to invoke the 'magic loophole'—there is enough energy available in the excited state.)

• **Question** The argument given provides a picture of a repulsive force. What can it say about attractive forces?

Answer This is almost always the first question asked. One cannot, in terms of the models produced so far, provide a satisfactory description of an attractive force. (Some would argue that the description given of the repulsive force is itself illusory; to some extent it is, but many feel it is justified because it enables them to gain valuable insight into the nature of forces without further knowledge of quantum mechanics.)

What else one says depends very much on who asks the question. Some are happy with the following popular classical analogy. Imagine a man in one boat and a woman in another. They have no means of propelling their boats, but happen to be supplied with boomerangs. How can they get together?

By throwing a boomerang away from the man, the woman would experience a reaction force from the boomerang towards the man. The boomerang could then circle round and approach the man from behind, and on reaching him, could exert a force on him towards the woman!

Some people do not like this analogy. It does, however, have two features that make it useful:

– It is thought-provoking and memorable, even as a piece of classical physics.
– It involves spin, and although this takes us beyond the scope of this introductory paper, it is now known that the force-carrying particles—the photon, the intermediate vector bosons Z and W, and the gluon—all have an intrinsic angular momentum or spin.

• **Question** The picture in figure 4 looks unacceptably jerky. We know that two electrons approaching each other from a large distance would repel each other gently at first, then gradually more strongly as their separation decreased. Can the exchange model of forces accommodate this?

Answer Yes, by imagining not one but millions of exchanged photons passing from one electron to another. Photons that travel a long way have less momentum and exert a weaker force; see next question.

• **Question** The (Coulomb) inverse square law of electrostatic repulsion (first published by the English 'chemist' Joseph Priestley [6]) says that the force still exists at very large separations. How are we to imagine a virtual photon travelling large distances within the time limit imposed by the Uncertainty Principle?

Answer Let us imagine an electron emitting a virtual photon of very low (almost zero) energy. The amount by which energy conservation is violated will be very small (almost zero); there is, in principle, no limit to how low the frequency of electromagnetic radiation can be—by moving a charge to and fro between two points with a period of billions of years, one generates virtual photons of almost zero energy! Such a virtual state can, according to the Uncertainty Principle, only exist for a time

$$\frac{h}{4\pi \times (\text{almost zero})}.$$

But this is an almost infinite time, during which a photon, travelling at the speed of light, could travel an almost infinite distance (delivering the gentlest of nudges)!

• **Question** I can see that we are getting a nice way of visualizing forces between charged particles. Can this way of looking at things contribute to our understanding of nuclear forces, which are different from electrical forces in being of very short range (a few times 10^{-15} m)?

Answer This is something that **Master** has promised to deal with!

Nuclear forces

Master The key point in enabling the *exchange* picture to provide a description of a force extending to infinity was that a photon can, in principle, have an energy of (more or less) zero.

If we imagine a force (or interaction) mediated by the passage of a material particle (one with a non-zero rest mass m), then the energy violation associated with just producing it must be at least mc^2. By the Heisenberg Uncertainty Principle this virtual state cannot exist for longer than $h/4\pi mc^2$. This is a **finite** time during which the particle can only travel a **finite** distance. The farthest the particle could conceivably go in this time is the speed of light times this time—$h/4\pi mc$. The range of the interaction is limited to this distance because if the particle it is *trying to interact with* is farther away, the exchange particle cannot reach it in the time it has available.

So we have an expression for the range R of an interaction:

$$R = \frac{h}{4\pi mc}.$$

This is a remarkable formula. It gives the range R of an interaction in terms of:

- Planck's constant h, the fundamental constant of quantum mechanics.
- The speed of light c, another fundamental constant of nature, one that is at the heart of relativity.
- The mass m of the exchanged (or *carrier*) particle. This appears in as simple a way as one could hope for: since it is on the bottom of the fraction, the larger m is, the shorter the range R of the force. This is intuitively reasonable—one can throw a golf ball further than a cannonball!

So now we have a way of visualizing short-range interactions as well as long-range ones!

Pupil Now that looks very interesting because, for nuclear interactions, we know the range R—it has been measured to be about 10^{-15} m. If we substitute this number into our formula for the range we can estimate the mass of the particle that this model says should be the *carrier* of the nuclear interaction.

Let me do it straight away! I know that $h = 6.63 \times 10^{-34}$ J s and $c = 3 \times 10^8$ m s^{-1}. So

$$m \approx \frac{6.63 \times 10^{-34}}{4\pi \times 3 \times 10^8 \times 10^{-15}} = 0.18 \times 10^{-27} \text{ kg}.$$

The mass of the proton is 1.673×10^{-27} kg. So this simple-minded estimate based on the exchange model of forces would suggest a virtual particle of rest mass roughly 1/9 that of the proton. What does this tell me?!

Master When this idea was first put forward by the Japanese physicist Yukawa in 1934, no particle with a mass anywhere near this value was known.

But one could have been tempted to speculate as follows: **if** the exchange model is good then the virtual particle with a mass of roughly 1/9 that of the proton might actually be capable of a real existence. After all, the model began, in the knowledge of the existence of real photons, by postulating the existence of virtual photons!

Amazingly, in 1947, such a particle was discovered in Bristol by Powell and collaborators. It is called the pion and is represented by the Greek symbol π. There are three pions—one positive (π^+), one negative (π^-) and one neutral (π^0). Apart from having a mass of 1/7 that of the proton (not far from 1/9), the pion has the right properties. In particular, when it is made to interact with protons, it does so strongly, as one would want for a particle which, in its virtual state, is supposed to hold protons in a nucleus, overcoming the electrical repulsion they exert on each other.

So, strange though the argument has been, we cannot but marvel at the power of quantum mechanics. (Yukawa was awarded the Nobel Prize for Physics in 1949, Powell in 1950.)

Pupil That is absolutely breathtaking! Let me recall the main points of the argument:

(1) From two experiments—the two-slit experiment with electrons and the photoelectric effect—I establish wave–particle duality with its two basic formulae:

$$p = \frac{h}{\lambda} \quad \text{and} \quad E = hf.$$

(2) Although I do not understand the mysterious quantum waves as I would like, I assume that, whatever properties other waves share, the quantum waves share too—beats, resonance, etc.

(3) If I consider measuring the frequency of a sung note, I discover an important point: I cannot measure the frequency to any desired accuracy. The accuracy I can achieve depends on how long I take to make the measurement. Put the other way round, to achieve an accuracy Δf, I need a time longer than a certain amount that depends inversely on Δf—the smaller Δf is (the better I determine the frequency), the longer I need.
This is the crucial argument, because, once it is accepted, one assumes the same argument holds for all waves, including quantum waves.

(4) The quantum version of this (the Heisenberg Uncertainty Principle), which chooses to talk in terms of energy E rather than frequency f (justified by $E = hf$ from above), states that to measure energy with an accuracy ΔE, one needs a time longer than $h/4\pi\,\Delta E$. Since the ΔE is on the bottom of the fraction, the better we determine E by getting ΔE small, the longer we need for the measurement.
(I have to accept that the 4π comes in from a slightly more rigorous argument **based on exactly the same physics, wave–particle duality.**)

(5) A consequence of the Heisenberg Uncertainty Principle is that one cannot exclude the possibility of processes that violate energy conservation by amounts $E_{\text{violation}}$ for times shorter than $h/4\pi\,E_{\text{violation}}$—the *magic loophole*.

(6) One such possibility is the emission of a *virtual photon*, for example, from an electron. If this virtual photon is absorbed by another electron, within the time limit imposed by the Uncertainty Principle, **in such a way that the total final energy equals the total initial energy**, then we have a model for how electrons exert forces on each other.

(7) Taking things one step further, we can show that to describe a short-range force like the nuclear force, one would need a virtual particle with a mass of about 1/9 of the proton mass, m_{p}. The pion, with a mass of about $m_{\text{p}}/7$, has been discovered.

(8) As was hinted earlier, things have moved on. We now know that protons, neutrons and pions are made of more fundamental particles called *quarks*. The strong force is the force between quarks and the corresponding force-carrying particle has been named the *gluon*. Nevertheless, at energies of the order of 1 GeV, it is more insightful to describe many interactions in terms of the *pion exchange* than to try to invoke quarks and gluons.

I think I am beginning to appreciate why modern physics generates so much excitement!

Master This is just the beginning. The rules we have learnt can describe many other phenomena. Also, no phenomena involving particles are known to violate the rules of quantum mechanics and relativity. This is true not only in the realms of what one might call *cosmic* physics (astronomy, cosmology, particle physics) but also *terrestrial* physics (electronics, biochemistry, material science, etc). Quantum mechanics and relativity constitute the best model we have for describing the behaviour of particles, and, since electrons are particles, this model is the seed from which our high-technology industries have grown.

But beware! Do not get carried away into thinking that this is the end of the line. There are lots of things we don't know! For example, we cannot even calculate the mass of the electron or proton, or any other particle, from our quantum mechanics and relativity.

Also, remember that the aims of science are to make discoveries about the world, and to describe these discoveries in terms of models. These models cannot be proved right because one cannot do all possible experiments. (The so-called *Theories of Everything* that we hear about on popular science programmes are not physical realities but articles of faith, based on the 'belief' that the world we address with our experiments is a manifestation of deep cosmic mathematics from which all physical reality stems. People who take this point of view refer to themselves as 'platonists', because Plato—in his theory of 'ideas' or 'forms'—taught that we can only have unreliable opinions about the world we perceive with our senses, that true knowledge can only be found in his 'world of ideas', which is accessed through the mind or the soul, as opposed to the body. The logical truths of mathematics would be included in Plato's world of ideas. We are now hovering at the metaphysical edge of science!)

A more humble and realistic standpoint is to picture scientific advance as an ever-growing island of knowledge and understanding in a possibly infinite sea of ignorance: the more it grows, the longer the boundary between the two!

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