A PRACTICAL INTRODUCTION TO $E = mc^2$

In 1932, Cockroft and Walton accelerated protons to energies of a few hundred keV and directed them onto a lithium target. They observed the reaction

$$_1H^1 + {}_3Li^7 \longrightarrow {}_2He^4 + {}_2He^4$$

The energies of the back to back α particles were measured in a cloud chamber to be about 8.5 meV each. (click here) (see 2008 Gill, the first transmutation of one nucleus into another using accelerated protons) This is huge compared to the energies of accelerated protons: kinetic energy is clearly not conserved.

Since the masses of all the nuclei involved are known, this provides an interesting way to examine the relationship between energy and mass.

IS ENERGY CONSERVED?

KE of proton ($_1H^1$) = few hundred keV
KE of ($_3Li^7$) = 0
KE of alphas ($_2He^4$) = 2 x 8.5 MeV

Observe: about 17 MeV is gained

IS MASS CONSERVED?

Mass of proton ($_1H^1$) = 1.0078 amu
Mass of ($_3Li^7$) = 7.0160 amu
Mass of alphas ($_2He^4$) = 2 x 4.0026 amu

Observe: about 0.0186 amu (or $0.0186 \times 1.66 \times 10^{-27}$ kg) are lost.

So the reaction

$$p + {}_3Li^7 \longrightarrow {}_2He^4 + {}_2He^4$$

releases 17 Mev of energy at the expense of $0.0186 \times 1.66 \times 10^{-27}$ kg.

Now, $1\text{eV} = 1.6 \times 10^{-19}$J

So, the 'exchange rate' in SI units is given by

$$\frac{\text{KE gained}}{\text{Mass lost}} = \frac{17\text{ MeV}}{3.088 \times 10^{-29}\text{kg}} \times \frac{(1.6 \times 10^{-13}\text{J})}{1\text{MeV}} = 8.81 \times 10^{16}\text{ J/kg} = (3 \times 10^8\text{ m/s})^2 = c^2$$

In the process

$$_1H^1 + {}_3Li^7 \longrightarrow {}_2He^4 + {}_2He^4 + 17\text{ MeV}$$

we can say that energy is conserved if we think of **mass x c^2** as a form of energy. In nuclear physics, this is roughly a 1% effect.

The same has been found to be true in all nuclear reactions. This leads to a new single conservation law, combining the old laws of conservation of energy and conservation of mass:

Initial *final*

Σ (KE + mc^2) + Σ (KE + mc^2)

NB This is a rule that works – that is, it enables one to calculate what happens – but it is not compatible with Newtonian physics.

Something new is needed. In **relativity, mass x c^2** is a form of energy.

Comment: this rule also applies to chemical reactions such as metabolism. However, here it is only a 10^{-8} effect which explains why violations of the old laws of energy and mass conservation were not discovered earlier.