Measurement of mass

In classical physics the mass of a particle can be determined from its energy, E, and its momentum, p:

$$E = \frac{p^2}{2m} \implies m = \frac{p^2}{2E}$$

The same is true for particles moving with a relativistic velocity, but we must use the correct formula:

$$E^2 = p^2c^2 + m^2c^4 \implies m = \sqrt{\frac{E^2 - p^2c^2}{c^4}}$$

The equation $E^2 = p^2c^2 + m^2c^4$ has two interesting limits:

- For a particle at rest ($p = 0$), it reduces to $E = mc^2$.
- For a highly relativistic particle, $m^2c^4 \ll p^2c^2$, it reduces to $E = pc$ (which holds true for photons).

It is also interesting to show that $E^2 = p^2c^2 + m^2c^4$ reduces to $E = mc^2 + \frac{p^2}{2m}$ (rest energy plus non-relativistic kinetic energy) when p is small:

$$E^2 = p^2c^2 + m^2c^4 \qquad \implies \qquad \begin{aligned} E &= mc^2\big(1 + \frac{p^2c^2}{m^2c^4}\big)^{1/2} = mc^2\big(1 + \frac{p^2}{m^2c^2}\big)^{1/2} \\ &= mc^2\big(1 + \frac{1}{2}\cdot\frac{p^2}{m^2c^2} + \ldots\big) \\ &\approx mc^2 + \frac{p^2}{2m} \qquad \text{(for small } p\text{)} \end{aligned}$$