Problem

Show that in the 2-body decay of a pion at rest

$$\pi \longrightarrow \mu \, \nu$$

the momentum p imparted to the decay products is about $30 \, MeV/c$.

Solution

By energy conservation

$$\begin{aligned} m_\pi c^2 &= E_\mu + E_\nu \\ &= \sqrt{p^2c^2 + m_\mu^2 c^4} + pc \end{aligned}$$

Re-arranging and squaring

$$m_\pi^2 c^4 - 2m_\pi p c^3 + p^2c^2 = p^2c^2 + m_\mu^2 c^4$$

Hence

$$2m_\pi p c^3 = (m_\pi^2 - m_\mu^2)c^4$$

and

$$p = \frac{(m_\pi^2 - m_\mu^2)c}{2m_\pi}$$

But $m_\pi = 139.6 \, MeV/c^2$ and $m_\mu = 105.7 \, MeV/c^2$. So

$$p = \frac{139.6^2 - 105.7^2}{2 \cdot 139.6} MeV/c \approx 30 \, MeV/c$$

When a charged pion stops in a bubble chamber and decays, the $30 \, MeV/c$ $\mu^\pm$ travels about a centimetre in hydrogen and then decays to an $e^\pm$.

If a $1 \, GeV/c = 1000 \, MeV/c$ $\pi^\pm$ decays in flight to a $\mu^\pm$, the maximum angle the μ can make to the π line of flight is

$$\tan^{-1} \frac{30}{1000} \approx 1.72^\circ$$